



Fractions Progression – A Look at Fractions in K-6

February 2015

"A high percentage of U.S. students lack the conceptual understanding of fractions even after studying fractions for several years; this in turn, limits students' ability to solve problems with fractions and learn and apply computational procedures involving fractions." NCEE 2010-4039 U.S. Department of Education

FOCUS ON FRACTIONS

The Arizona College & Career Ready Standards for Mathematics are composed of Domains. Clusters of standards make up each Domain. Within each grade level, major Clusters have been identified. Arizona Department of Education has specified that between 65% and 85% of instruction should focus on the major Clusters. In grades 3-6, all Clusters dealing with fractions are considered major clusters. Our students have consistently struggled with fraction concepts and often arrive in middle and high school without the necessary understanding to be successful with higher level concepts. This newsletter will look at the progression of fractions from kindergarten through sixth grade.

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Kindergarten Through 2nd Grade: Geometry Domain (G)

Building fraction sense begins in grade K through the Geometry (G) domains. In the Geometry domain, the standards in K-2 are found within supporting content clusters.

KINDERGARTEN STANDARDS – K.G.

- K.G.B.5.** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
- K.G.B.6.** Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

GRADE 1 STANDARDS 1.G.

1.G.A.3. Partition circles and rectangles into two and four equal shares, describe the shares using the words halves, fourths, and quarters, and use the phrases half of, fourth of, and quarter of. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

GRADE 1 EXAMPLES 1.G.

Students understand cutting a rectangle or circle in half makes two parts, and one part is smaller than two parts. They also cut rectangles and circles into four parts and understand that one fourth is smaller than one half.

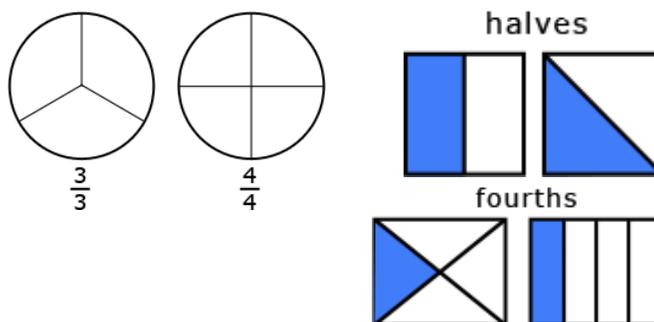


GRADE 2 STANDARDS 2.G.

2.G.A.3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

GRADE 2 EXAMPLES 2.G.

This standard introduces fractions in an area model. Students need experiences with different sizes, circles, and rectangles.



Resources

The standards & examples in this publication were copied from the AZCCR-M documents at www.azed.gov. Here are a few links containing additional information, including details on what each child will be expected to know and do in each grade and tips for parents:

<http://ccesa.az.gov/>

www.azed.gov/AzMERIT

<http://www.azed.gov/assessment/azsmp/assessmentitems/>

<http://achievethecore.org/>

<https://www.engageny.org/>

www.corestandards.org

www.pta.org/parentsguide

<http://www.azed.gov/standards-practices/files2012/05/rttt-implementation-plan-2-6-12.pdf>

www.theteachingchannel.org/



1st & 2nd Grade: Measurement & Data Domain (MD)

In first and second grade, the Measurement & Data standards that build fraction concepts (1.MD.A.2 & 2.MD.B.6) are part of a major content clusters.

GRADE 1 STANDARDS 1.MD.

1.MD.A.2. Express the length of an object as a whole number of length units by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

1.MD.B.3. Tell and write time in hours and half-hours using analog and digital clocks.

GRADE 1 EXAMPLES 1.MD.

Ask students to use multiple units of the same object to measure the length of a pencil. (How many paper clips will it take to measure how long the pencil is?)



The idea of 30 being "halfway" is difficult for students to grasp. Students can write the numbers from 0 - 60 counting by tens on a sentence strip. Fold the paper in half and determine that halfway between 0 and 60 is 30. A number line on an interactive whiteboard may also be used to demonstrate this.

"It is halfway between 8 o'clock and 9 o'clock. It is 8:30."



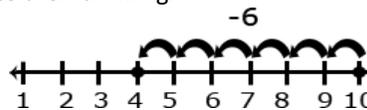
GRADE 2 STANDARDS 2.MD.

2.MD.B.6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

GRADE 2 EXAMPLES 2.MD.

Students represent their thinking when adding and subtracting within 100 by using a number line. An interactive whiteboard or document camera can be used to help students demonstrate their thinking.

$10 - 6 = 4$



3rd-5th Grade: Number & Operation - Fractions (NF)

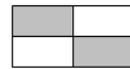
Number and Operation – Fractions (NF) becomes its own domain in 3rd grade. All NF standards are found within major content clusters for grades 3-5.

3.NF.A.1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

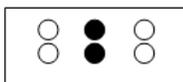
GRADE 3 EXAMPLES 3.NF.A.1.

Students express fractions as fair sharing, parts of a whole, and parts of a set. They need many opportunities to solve word problems that require fair sharing. To develop understanding of fair shares, students first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.

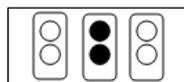
- Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
- Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?
- What fraction of the rectangle is shaded? How might you draw the rectangle in another way but with the same fraction shaded?



Solution: $\frac{2}{4}$ or $\frac{1}{2}$



Solution: $\frac{2}{3}$

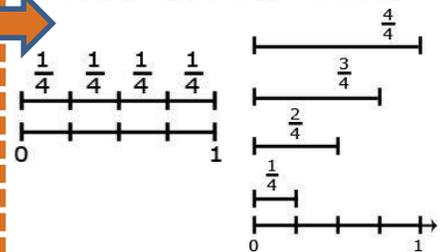


Solution: $\frac{1}{3}$

3.NF.A.2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.

- Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
- Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

GRADE 3 EXAMPLES 3.NF.A.2



GRADE 3 STANDARDS 3.NF. Continued

3.NF.A.3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

- Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
- Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
- Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.*
- Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

GRADE 3 EXAMPLES 3.NF.A.3.

An important concept when comparing fractions is to look at the size of the parts and the number of the parts.

- For example, $1/8$ is smaller than $1/2$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces.

Students recognize when examining fractions with common denominators, the wholes have been divided into the same number of equal parts. So the fraction $\frac{2}{6} < \frac{5}{6}$ with the larger numerator has the larger number of equal parts.

To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts are different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces.

GRADE 4 STANDARDS

4.NF.

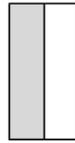
4.NF.A.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100). Students can use visual models or applets to generate equivalent fractions.

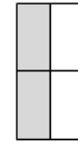
All the models show $1/2$. The second model shows $2/4$ but also shows that $1/2$ and $2/4$ are equivalent fractions because their areas are equivalent. When a horizontal line is drawn through the center of the model, the number of equal parts doubles and size of the parts is halved.

Students will begin to notice connections between the models and fractions in the way both the parts and wholes are counted and begin to generate a rule for writing equivalent fractions.

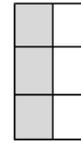
$1/2 \times 2/2 = 2/4.$



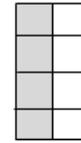
$\frac{1}{2}$



$\frac{2}{4} = \frac{2 \times 1}{2 \times 2}$



$\frac{3}{6} = \frac{3 \times 1}{3 \times 2}$



$\frac{4}{8} = \frac{4 \times 1}{4 \times 2}$

4.NF.A.2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

GRADE 4 EXAMPLES 4.NF.A.2.

Benchmark fractions include common fractions between 0 and 1 such as halves, thirds, fourths, fifths, sixths, eighths, tenths, twelfths, and hundredths.

Fractions can be compared using benchmarks, common denominators, or common numerators. Symbols used to describe comparisons include $<$, $>$, $=$.

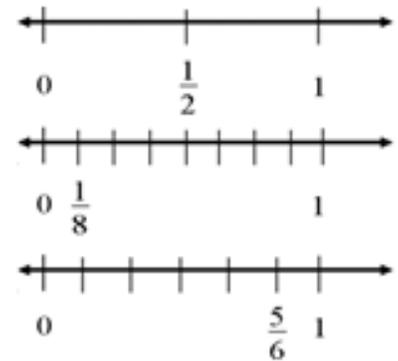
Fractions may be compared using $\frac{1}{2}$ as a benchmark.

Fractions with common denominators may be compared using the numerators as a guide.

$\frac{2}{6} < \frac{3}{6} < \frac{5}{6}$

Fractions with common numerators may be compared and ordered using the denominators as a guide.

$\frac{3}{10} < \frac{3}{8} < \frac{3}{4}$



GRADE 4 STANDARDS 4.NF. Continued

4.NF.B.3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

- Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples:* $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2\ 1/8 = 1 + 1/8 = 8/8 + 8/8 + 1/8$.
- Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
- Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

GRADE 4 EXAMPLES 4.NF.B.3.

A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as $2/3$, they should be able to decompose the non-unit fraction into a combination of several unit fractions.

- Fraction Example 1:

$$2/3 = 1/3 + 1/3$$

- Fraction Example 2:

$$1\ 1/4 - 3/4 = \square$$

$$4/4 + 1/4 = 5/4$$

$$5/4 - 3/4 = 2/4 \text{ or } 1/2$$

- Word Problem Example 1:

Mary and Lacey decide to share a pizza. Mary ate $3/6$ and Lacey ate $2/6$ of the pizza. How much of the pizza did the girls eat together?

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

- Word Problem Example 2:

Susan and Maria need $8\ 3/8$ feet of ribbon to package gift baskets. Susan has $3\ 1/8$ feet of ribbon and Maria has $5\ 3/8$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

4.NF.B.4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

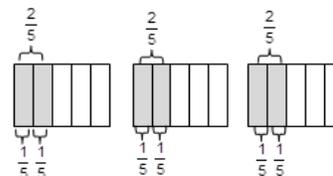
- Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.*
- Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)*
- Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

GRADE 4 EXAMPLES 4.NF.B.4.

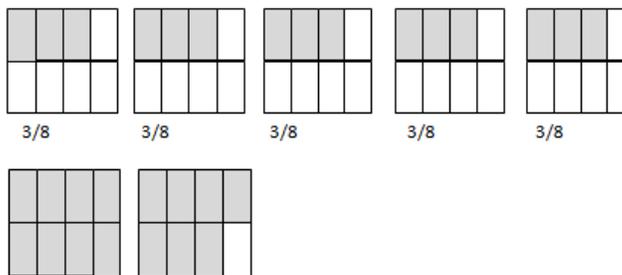
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

Examples:

$$3 \times (2/5) = 6 \times (1/5) = 6/5$$



If each person at a party eats $3/8$ of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?



$$3/8 + 3/8 + 3/8 + 3/8 + 3/8 = 15/8 = 1\ 7/8$$

4.NF.C.5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$. (Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general, but addition and subtraction with unlike denominators in general is not a requirement at this grade.)*

GRADE 4 EXAMPLES 4.NF.C.5.

Students may represent $3/10$ with 3 longs and may also write the fraction as $30/100$ with the whole in this case being the flat (the flat represents one hundred units with each unit equal to one hundredth). Students begin to make connections to the place value chart as shown in 4.NF.6.

GRADE 4

4.NF. Continued

4.NF.C.6. Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as $\frac{62}{100}$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.

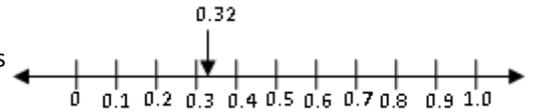
GRADE 4 EXAMPLES 3.NF.C.6.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

| | | | | | |
|----------|------|------|---|--------|------------|
| Hundreds | Tens | Ones | • | Tenths | Hundredths |
| | | | • | 3 | 2 |

Students use the representations explored in 4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$.

Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.

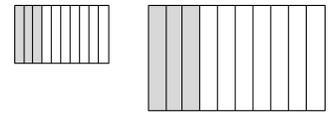


4.NF.C.7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.

GRADE 4 EXAMPLES 4.NF.C.7.

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases.

- Each of the models below shows $\frac{3}{10}$ but the whole on the right is much bigger than the whole on the left. They are both $\frac{3}{10}$ but the model on the right is a much larger quantity than the model on the left.



When the wholes are the same, the decimals or fractions can be compared.

Example:

- Draw a model to show that $0.3 < 0.5$. (Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.)



GRADE 5 STANDARDS 5.NF.

5.NF.A.1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)

GRADE 5 EXAMPLES 5.NF.A.1.

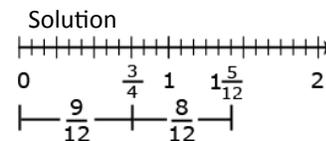
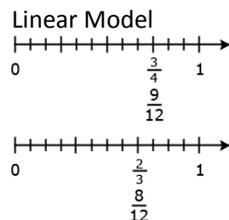
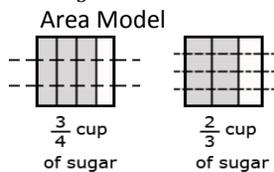
Examples:

- $\frac{2}{5} + \frac{7}{8} = \frac{16}{40} + \frac{35}{40} = \frac{51}{40}$
- $3\frac{1}{4} - \frac{1}{6} = 3\frac{3}{12} - \frac{2}{12} = 3\frac{1}{12}$

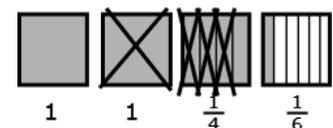
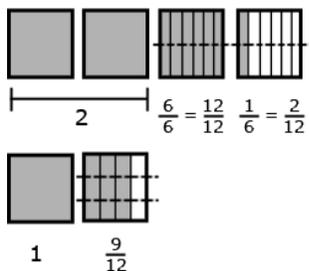
5.NF.A.2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

GRADE 5 EXAMPLES 5.NF.A.2.

- Jerry was making two different types of cookies. One recipe needed $\frac{3}{4}$ cup of sugar and the other needed $\frac{2}{3}$ cup of sugar. How much sugar did he need to make both recipes?



- This model shows $1\frac{3}{4}$ subtracted from $3\frac{1}{6}$ leaving $1 + \frac{1}{4} + \frac{1}{6}$ which a student can then change to $1 + \frac{3}{12} + \frac{2}{12} = 1\frac{5}{12}$.



- This diagram models a way to show how $3\frac{1}{6}$ and $1\frac{3}{4}$ can be expressed with a denominator of 12. Once this is accomplished, a student can complete the problem, $2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$.

GRADE 5 STANDARDS 5.NF. Continued

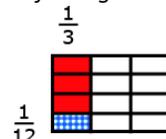
5.NF.B.7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division, but division of a fraction by a fraction is not a requirement at this grade.)

- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.*
- Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.*
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?*

GRADE 5 EXAMPLES 5.NF.B.7.

Knowing the number of groups/shares and finding how many/much in each group/share:

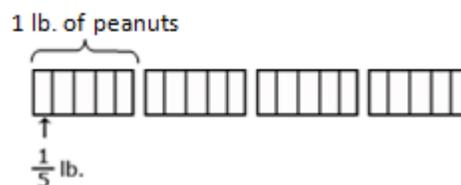
- Four students sitting at a table were given $1/3$ of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?



Knowing how many in each group/share and finding how many groups/shares:

- Angelo has 4 lbs of peanuts. He wants to give each of his friends $1/5$ lb. How many friends can receive $1/5$ lb of peanuts?

A diagram for $4 \div 1/5$ is shown below. Students explain that since there are five fifths in one whole, there must be 20 fifths in 4 lbs.



6th Grade: The Number System (NS)

The Number System (NS) is the domain that contains dividing fractions in Grade 6. The cluster (NS.A.) is a major content cluster for 6th grade. Dividing fractions should not be taught as the trick "Keep, Change, Flip." Instead, students develop the understanding that dividing by a number is the same as multiplying by its inverse.

GRADE 6 STANDARDS 6.NS.A

6.NS.A.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for $(2/3) \div (3/4)$ and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb. of chocolate equally? How many $3/4$ -cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?*

GRADE 5 EXAMPLES 5.NF.B.7.

- 3 people share $1/2$ pound of chocolate. How much of a pound of chocolate does each person get? (Solution: Each person gets $1/6$ lb. of chocolate.)
- Manny has $1/2$ yard of fabric to make book covers. Each book is made from $1/8$ yard of fabric. How many book covers can Manny make? (Solution: Manny can make 4 book covers.)
- Represent $1/2 \div 2/3$ in a problem context and draw a model to show your solution.

Context: You are making a recipe that calls for $2/3$ cup of yogurt. You have $1/2$ cup of yogurt from a snack pack. How much of the recipe can you make?

Explanation of Model:

- The first model shows $1/2$ cup.
- The second model shows $1/2$ cup and also shows $1/3$ cups horizontally.
- The third model shows $1/2$ cup moved to fit in only the area shown by $2/3$ of the model. $2/3$ is the new referent unit (whole).
- 3 out of the 4 squares in the $2/3$ portion are shaded. A $1/2$ cup is only $3/4$ of a $2/3$ cup portion, so you can only make $3/4$ of the recipe.

